

# S-wave $\pi^0$ Production in pp Collision in a OBE Model

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The total cross section for the  $pp \rightarrow pp\pi^0$  reaction at energies close to threshold is calculated using a covariant one-boson-exchange model, where a boson B created on one of the incoming protons is converted into a neutral pion on the second. The amplitudes for the conversion processes,  $BN \rightarrow N\pi^0$ , are taken to be the sum of s, u and t-channel pole terms. The main contributions to the primary production amplitude is due to an effective isoscalar  $\sigma$  meson pole in a t-channel, which is enhanced strongly due to offshellness. With this contribution included the model reproduces, both the scale and energy dependence of the cross section.

Key Words :  $\pi^0$  Production, Covariant OBE Model, Effective Two-pion exchanges.

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## I. INTRODUCTION

In a recent study of the  $pp \rightarrow pp\pi^0$  reaction at energies close to threshold, it is found that the angular distributions of the outgoing particles are isotropic, in agreement with the assumption that the reaction proceeds as a  $^{33}P_0 \rightarrow ^{31}S_0 s_0$  transition [1,2]. Several model calculations [3–6] of S-wave pion production, which are based on a single nucleon and a pion rescattering mechanism, under estimate the cross section by a factor of 3-5. Inspired by a study of  $\beta$ -decay in nuclei, which indicates that the axial charge of the nuclear system is enhanced by heavy meson exchanges and, by using a simple operator form of the NN potential, Lee and Riska [7] have shown that meson exchange currents could explain the scale of the cross section. Adding to the single nucleon and rescattering terms a Z-graph describing various heavy meson exchanges, Horowitz et al. [8] performed similar calculations based on explicit one-boson-exchange (OBE) model for the NN interaction as well as for the evaluation of meson exchange contributions.

The contribution from the rescattering term depends on the off-shell behavior of the  $\pi N \rightarrow \pi N$  scattering amplitude. There are several approaches based on field theoretical models which allow such extension to be made but the results are model dependent. In the traditional phenomenological treatment [3–6], the off mass-shell and on mass-shell amplitudes are of the same order of magnitude. Recently, several groups [9–13] have shown that the off mass-shell  $\pi N$  rescattering amplitude is enhanced strongly with respect to the on mass-shell amplitude. Hernandez and Oset [9] by applying current algebra and PCAC constraints argue that this enhancement may bring the calculated cross section for the  $pp \rightarrow pp\pi^0$  reaction into agreement with experiment. However based on detailed momentum-space calculations, Hanhart et al. [10] conclude that the enhancement of the rescattering amplitude due to offshellness falls a bit too short to explain the scale of the  $pp \rightarrow pp\pi^0$  cross section. Park et al. [11], Cohen et al. [12] and Sato et al. [13] have applied a chiral perturbation theory ( $\chi$ PT), including chiral order 0 and 1 Lagrangian terms. They have shown that the off mass-shell  $\pi N$  elastic scattering amplitude is enhanced considerably but has an opposite sign with respect to the on mass-shell amplitude. Because of this difference in sign, the rescattering term and the Born term contributing to the  $pp \rightarrow pp\pi^0$  reaction interfere destructively, making the theoretical cross sections much smaller than experimental values and thus suggesting a significant role for heavy meson exchanges in  $\pi^0$  production.

In the present work we report on a covariant OBE model calculations based on the mechanism depicted in Fig. 1, where a virtual boson B ( $B=\pi, \sigma, \eta, \rho, \omega, \delta \dots$ ) created on one of the incoming nucleons, is converted into a  $\pi^0$  meson on the second via a  $BN \rightarrow \pi^0 N$  conversion process. The half off mass-shell amplitudes for the conversion processes, hereafter denoted by  $T_{BN \rightarrow \pi^0 N}$ , are taken as the sum of three terms corresponding to the s, u and t-channels displayed in Fig. 2. One important aspect of this mechanism is that at the  $\pi^0$  production threshold, the transferred 4-momentum is space-like,  $q^2 = -3.3 \text{ fm}^{-2}$ . This is very much the same kinematic as occurring in  $\pi^0$  electroproduction through vector meson exchanges, and we apply a formalism similar to the one applied for electroproduction amplitudes [14] calculations. The formalism to be applied is consistent with the OBE picture of the nuclear force and accounts for relativistic effects, crossing symmetry, energy dependence and nonlocality of the hadronic interactions.

We start with the assumption that the pion production in  $pp \rightarrow pp\pi^0$  can be described by the graphs of Fig. 3. The diagrams 3a and 3b represent nucleon pole terms where the neutral pion is produced on a nucleon line. The diagram 3c depicts a pion production occurring on an internal meson line. By applying s-channel unitarity to NN elastic scattering in the energy region below two-pion production threshold, it can be shown that mechanisms as such should contribute to the production process also [15]. Schütz et al. [16] reported on  $\sigma$  and  $\rho$  meson t pole contributions to  $\pi N$  scattering. Their model was based on correlated  $2\pi$  exchange and constrained by using quasiempirical information about the  $N\bar{N} \rightarrow \pi\pi$  amplitudes. Hanhart et al. [10] used the T-matrix obtained in this model to calculate the rescattering term for  $pp \rightarrow pp\pi^0$ . More recently, van Kolck et al. [17] considered other short range meson exchange mechanisms like  $\rho \rightarrow \omega\pi$  and  $\delta \rightarrow \eta\pi^0$ . To keep our model calculations consistent with the OBE picture of Machleidt [19] we limit our discussions to contributions from  $\sigma$ ,  $\delta$  and  $\rho$  poles. In variance with previous calculations [16,17] we write these vertices in a more general form and determine the relevant coupling constants by applying the Adler's consistency conditions and/or fitting to some observables. We shall demonstrate below that a  $\sigma$ -meson t pole term which accounts *effectively* for isoscalar-scalar two-pion exchanges contributes significantly to  $\pi^0$  production. With this contribution included the model explains the cross-section data for the  $pp \rightarrow pp\pi^0$  reaction at threshold.

The present article is organized as follows. In Sect. II we present details of the formalism and model parameters. The evaluation of the  $T_{\pi^0 N \rightarrow \pi^0 N}$ ,  $T_{\eta p \rightarrow \pi^0 p}$  and  $T_{\rho p \rightarrow \pi^0 p}$  conversion amplitudes require knowledge of the  $\sigma\pi\pi$ ,  $\delta\eta\pi$  and  $\rho\omega\pi$  vertices. These are deduced in subsections B-D. In Sect. III we write the S-wave production amplitude in a form suitable for numerical calculations. Model predictions are given in Sect. IV. At energies close to threshold, final state interactions (FSI) influences the energy dependence of the calculated cross section [20]. These and initial state interactions (ISI) corrections are introduced in Sect. V where comparison with data is to be made. We conclude and summarize in Sect. VI.

## II. THE MODEL

We use the following Lagrangian interaction:

$$L = \frac{f_{\pi NN}}{m_\pi} \bar{N} \gamma^5 \gamma^\mu \partial_\mu \pi \tau N + \frac{f_{\eta NN}}{m_\eta} \bar{N} \gamma^5 \gamma^\mu \partial_\mu \eta N + g_{\sigma NN} \bar{N} \sigma N + \\ g_{\rho NN} \bar{N} \left( \gamma^\mu + \frac{\kappa_V}{2M} \sigma^{\mu\nu} \partial_\nu \right) \tau \rho_\mu N + g_{\omega NN} \bar{N} \gamma^\mu \omega_\mu N + g_{\delta NN} \bar{N} \tau \delta N , \quad (2.1)$$

with obvious notations. This expression includes terms identical to the ones used by Machleidt et al. [19] to fit NN elastic scattering data in the energy region 0-420 MeV. Here as in Ref. [19] pseudovector couplings are assumed for the pseudoscalar mesons and the  $\omega$  tensor coupling is taken to be zero. All of the coupling constants, meson masses and cut off parameters are taken from Table A.2 of Ref. [19]; their potential C parameter set.

To calculate the transition amplitude we assume that the reaction is dominated by the mechanism depicted in Fig. 1 and write the primary production amplitude as,

$$M^{(in)} = \sum_B [T_{BN \rightarrow \pi^0 N}(p_4, k; p_2, q) G_B(q) S_{BNN}(p_3, p_1)] + [1 \leftrightarrow 2; 3 \leftrightarrow 4] . \quad (2.2)$$

Here  $p_i$ ,  $q$  and  $k$  are 4-momenta of the i-th nucleon, the exchanged boson and the outgoing pion. The sum runs over all possible B bosons that may contribute to the process. To be consistent with the OBE picture of the NN force [19], we include exchange contributions from all of the  $\pi$ ,  $\eta$ ,  $\sigma$ ,  $\rho$ ,  $\omega$  and  $\delta$  mesons. The bracket  $[1 \leftrightarrow 2; 3 \leftrightarrow 4]$  stands for a similar sum with the  $p_1$ ,  $p_3$  and  $p_2$ ,  $p_4$  momenta interchanged. In Eqn. 2.2,  $T_{BN \rightarrow \pi^0 N}$  represents the conversion amplitude corresponding to the  $BN \rightarrow \pi^0 N$  process. The quantities  $G_B(q)$  and  $S_{BNN}(p_3, p_1)$  are the propagator and source function of the meson exchanged, respectively. Throughout this work we use covariant expressions for the meson propagators and form factors as defined in Ref. [19]. The source functions for scalar, pseudoscalar and vector mesons are

$$S_{SNN}(p_1, p_3) = \bar{u}(p_3) I u(p_1) F_S(q) , \quad (2.3)$$

$$S_{PNN}(p_1, p_3) = \bar{u}(p_3) \gamma^5 I u(p_1) F_P(q) , \quad (2.4)$$

$$S_{VNN}^\mu(p_1, p_3) = \bar{u}(p_3) \left[ \gamma^\mu F_V^{(1)}(q_{13}^2) + i\sigma^{\mu\nu} q_\nu F_V^{(2)}(q_{13}^2) \right] I u(p_1) , \quad (2.5)$$

where  $u(p)$  stands for a nucleon Dirac spinor;  $I$  is the appropriate isospin operator and  $p_3 = p_1 - q$  is the final nucleon momentum. The functions  $F_S(q)$  and  $F_P(q)$  are source form factors for scalar and pseudoscalar mesons. For vector mesons there are two such quantities  $F_V^{(1)}$  and  $F_V^{(2)}$  representing vector and tensor form factors, the analogous of the nucleon electromagnetic form factors. In the calculations to be presented below all source form factors are taken in the form [19],

$$F_B(q^2) = g_{BNN} f_B(q); \quad f_B(q) = \frac{\Lambda_B^2 - m_B^2}{\Lambda_B^2 - q^2} . \quad (2.6)$$

Albeit, the propagators and source functions are rather well determined from fitting NN scattering data [19]. Thus to a large extent, the model success in explaining cross section data for the  $pp \rightarrow pp\pi^0$  reaction depends on how well the conversion amplitudes  $T_{BN \rightarrow \pi^0 N}$  are calculated. The relative importance of the various exchange contributions depend upon the off mass shell behavior of these amplitudes. For example, though very small on mass-shell, the amplitude for  $\pi^0 p \rightarrow \pi^0 p$  process is strongly enhanced off mass-shell [9,10] giving rise to a dominant contribution to the production rate.

### A. Nucleon pole s and u channel amplitudes

In this subsection we write expressions for s and u-channel nucleon pole terms (diagrams 2a and 2b). These are common to all of the various meson exchanges. We call  $p$  and  $p'$  the momenta of incoming and outgoing nucleon,  $q$  and  $k$  those of the boson B and the pion produced, respectively. Let  $s = (p' + k)^2$  be the total energy and

$$\Delta_N(x) = \frac{i}{x^2 - M^2 + i\epsilon} . \quad (2.7)$$

Using the vertices of the Lagrangian, Eqn. 1, and the usual Feynman rules the contributions from diagram 2a with B designating a scalar (S), a pseudoscalar (P) and a vector meson (V) are respectively,

$$T_S^{(s)} = -\frac{g_{SNN} f_{\pi NN}}{m_\pi} f_S(q) f_\pi(k) \bar{u}(p') \gamma^5 \not{k} \frac{\not{p} + \not{k} + M}{(p' + k)^2 - M^2} u(p) , \quad (2.8)$$

$$T_P^{(s)} = i \frac{f_{PNN} f_{\pi NN}}{m_P m_\pi} f_P(q) f_\pi(k) \bar{u}(p') \gamma^5 \not{k} \frac{\not{p} + \not{k} + M}{(p' + k)^2 - M^2} \gamma^5 \not{q} u(p) , \quad (2.9)$$

$$T_V^{(s)} = -\frac{g_{VNN} f_{\pi NN}}{m_\pi} f_V(q) f_\pi(k) \bar{u}(p') \gamma^5 \not{k} \frac{\not{p} + \not{k} + M}{(p' + k)^2 - M^2} \left[ \gamma^\mu + \frac{\kappa_V}{2M} \sigma^{\mu\nu} q_\nu \right] u(p) , \quad (2.10)$$

where  $\not{q} = a^\mu q_\mu$ . After some algebra one obtains,

$$T_S^{(s)} = i \frac{g_{SNN} f_{\pi NN}}{m_\pi} f_S(q) f_\pi(k) \bar{u}(p') \gamma^5 \left[ (s - M^2) + 2M \not{k} \right] u(p) \Delta_N(s) , \quad (2.11)$$

$$T_P^{(s)} = (2M)^2 \frac{f_{PNN} f_{\pi NN}}{m_P m_\pi} f_P(q) f_\pi(k) \bar{u}(p') \left[ -\left( \frac{s - M^2}{2M} \right) + \not{k} \left( \frac{s - M^2 + 4M^2}{4M^2} \right) \right] \Delta_N(s) u(p) , \quad (2.12)$$

$$T_V^{(s)} = i \frac{g_{VNN} f_{\pi NN}}{m_\pi} \Delta_N(s) f_V(q) f_\pi(k) \bar{u}(p') \gamma^5 \left[ \gamma^\mu (s - M^2) + 4k^\mu - 2M \gamma^\mu \not{k} \right] \left[ 1 + \left( \frac{\kappa_V}{2M} \right) \not{q} \right] u(p) . \quad (2.13)$$

The contributions from the u-channel (diagram 2b) are obtained easily by replacing the total energy  $s$  in Eqns. 11-13 with  $u = (p - k)^2$ . Likewise, for a vector meson the u-channel pole term is

$$T_V^{(u)} = -i \frac{g_{VNN} f_{\pi NN}}{m_\pi} \Delta_N(s) f_V(q) f_\pi(k) \bar{u}(p') \gamma^5 \gamma^\mu \left( 1 - \frac{\kappa_V}{2M} \not{q} \right) \left[ u - M^2 + 2M \not{k} \right] u(p) . \quad (2.14)$$

## B. Evaluation of $T_{\pi^0 N \rightarrow \pi^0 N}$

We now turn to calculate the amplitude  $T_{\pi^0 N \rightarrow \pi^0 N}$ . In this case in addition to s and u-channel nucleon pole terms, a neutral pion can be formed on an internal  $\sigma$  meson line. These terms are displayed graphically in Fig. 4. The contribution from graph 4c depends upon the  $\sigma\pi\pi$  meson vertex. There is no information about this coupling from traditional analyses of NN elastic scattering [19]. In order to determine this vertex we apply some physics beyond the OBE picture of the NN interactions. Quite generally we write an effective  $\sigma\pi\pi$  vertex in the form,

$$V_{\sigma\pi\pi}(k, q) = m_\sigma \left( g_{0\sigma} + \frac{q^2 + k^2}{m_\sigma^2} g_{1\sigma} + \frac{(q - k)^2}{m_\sigma^2} g_{2\sigma} \right) , \quad (2.15)$$

where  $g_{0\sigma}, g_{1\sigma}, g_{2\sigma}$  are (as yet unknown) constants. This is a slightly generalized Weinberg-type low energy expansion [21] allowing for three rather than one different constants. Taking the appropriate s and u-channel contribution from Eqn. 12 and adding the contribution from a  $\sigma$  pole in a t-channel one obtains,

$$T_{\pi^0 p \rightarrow \pi^0 p} = i \left( \frac{2M f_{\pi NN}}{m_\pi} \right)^2 f_\pi(q) f_\pi(k) \bar{u}(p') \left[ \not{k} \left( \frac{1}{s - M^2} - \frac{1}{u - M^2} \right) - \frac{1}{M} \right] u(p) + i g_{\sigma NN} \frac{f_\sigma(q - k)}{(q - k)^2 - m_\sigma^2} V_{\sigma\pi\pi} \bar{u}(p') u(p) . \quad (2.16)$$

Now, in order to determine the vertex constants  $g_{0\sigma}, g_{1\sigma}$  and  $g_{2\sigma}$  we require that the conversion amplitude obeys the three Adler's consistency conditions [22,23]. These are

$$T_{\pi^0 p \rightarrow \pi^0 p}(k = q = 0) = -i \frac{\sigma_{\pi N}(0)}{F_\pi^2} , \quad (2.17)$$

$$T_{\pi^0 p \rightarrow \pi^0 p}(k = 0, q^2 = m_\pi^2) = 0 , \quad (2.18)$$

$$T_{\pi^0 p \rightarrow \pi^0 p}(k^2 = m_\pi^2, q = 0) = 0 , \quad (2.19)$$

where  $\sigma_{\pi N}(0)$  is the well known  $\pi N$   $\sigma$ -term [24] and  $F_\pi$  the pion radiative decay constant. These three conditions yield only two relations amongst the constants, which are

$$g_{0\sigma} = \frac{\sigma_{\pi N}(0)m_\sigma}{F_\pi^2 g_{\sigma NN} f_\sigma(0)} , \quad (2.20)$$

$$g_{0\sigma} = \left( \frac{m_\pi}{m_\sigma} \right)^2 (g_{1\sigma} + g_{2\sigma}) . \quad (2.21)$$

Furthermore, we recall that the on mass shell  $\pi^0 p$  S-wave elastic scattering amplitude is parameterized according to [25,26],

$$F = a^+ + b^+ \mathbf{k} \cdot \mathbf{k} , \quad (2.22)$$

with  $a^+$  being the isospin even  $\pi^0 p$  scattering length. To obtain a third relation we may require that the amplitude Eqn. 17 reproduces  $F$  at  $\mathbf{k} = 0$ , i.e.,

$$T_{\pi^0 p \rightarrow \pi^0 p}(k^2 = m_\pi^2, \mathbf{k} = 0; q^2 = m_\pi^2, \mathbf{q} = 0) = i4\pi \left( 1 + \frac{m_\pi}{M} \right) a^+ . \quad (2.23)$$

This yields,

$$g_{0\sigma} + 2\left(\frac{m_\pi}{m_\sigma}\right)^2 g_{1\sigma} = \frac{-1}{g_{\sigma NN}} \left[ 4\pi \left( 1 + \frac{m_\pi}{M} \right) m_\sigma a^+ - f_{\pi NN}^2 \left( \frac{m_\sigma}{M} \right) \left( \frac{4M^2}{4M^2 - m_\pi^2} \right) \right] . \quad (2.24)$$

We now may use  $\sigma_{\pi N}(0)$ ,  $F_\pi$ ,  $a^+$  and  $g_{\sigma NN}$  as input to evaluate the vertex constants. The value of the quantity  $\sigma_{\pi N}(0)$  is related to matrix elements of the operator  $m_q q \bar{q}$  in the proton, where  $m_q$  stands for the mass of the proton quark constituents. This quantity can be calculated from the baryon spectrum [27]. To leading orders in the quark masses one finds to order  $O(m_q^{3/2})$  that  $\sigma_{\pi N}(0) = 26$  MeV. Including  $O(m_q^{3/2})$  contributions and estimate  $O(m_q^2)$  ones yields,

$$\sigma_{\pi N}(0) = \frac{35 \pm 5}{1 - y} \text{ MeV} , \quad (2.25)$$

with  $y$  being a measure of the strange quark content in the proton. An even higher a value is obtained from the isospin even  $\pi N$  scattering amplitude at the Cheng-Dashen point, which can be determined experimentally using the low-energy theorem of current algebra. This gives [27] a value  $\sigma_{\pi N}(0) = 45 \pm 8$  MeV. Equation 2.25 suggests a value  $\sigma_{\pi N}(0) = 35$  MeV for  $y = 0$ , in keeping with the OZI rule [28]. With this value and taking a radiative decay constant  $F_\pi = 93.5$  MeV [29]; isospin even  $\pi^0 p$  scattering length  $a^+ = (-0.010 \pm 0.003)m_\pi^{-1}$  [25,26], and the  $\sigma NN$  coupling  $g_{\sigma NN}^2/4\pi = 8.03$  [19] we can now solve Eqns. 2.20, 2.21, 2.24 to extract the values,

$$g_{0\sigma} = 0.22 \pm 0.04, \quad g_{1\sigma} = -0.82 \pm 0.42, \quad g_{2\sigma} = -2.62 \pm 0.7 . \quad (2.26)$$

With these constants Eqns. 2.16 predicts  $b^+ = (-0.068 \pm 0.015)m_\pi^{-3}$  in close agreement with the experimental value  $b_{exp}^+ = -(0.088 \pm 0.014)m_\pi^{-3}$  quoted by Koch [25]. Thus the amplitude Eqn. 2.16 on the mass shell reproduces the correct  $\mathbf{k} \cdot \mathbf{k}$  dependence of the  $\pi^0 p$  elastic scattering amplitude, Eqn. 2.22, also.

It is difficult to ascertain that the amplitude Eqn. 2.16 does reproduce the off mass shell behavior correctly. However we may confront Eqn. 2.16 with predictions from other approaches. For example, taking the residue of the amplitude, Eqn. 2.16, at the  $\sigma$  pole  $(q - k)^2 = m_\sigma^2$ , one obtains an effective  $\sigma \pi \pi$  coupling

$$g_{\sigma \pi \pi}^{eff} = V_{\sigma \pi \pi}(q^2 = m_\pi^2, k^2 = m_\pi^2, (q - k)^2 = m_\sigma^2) = (2.5 \pm 0.9)m_\sigma , \quad (2.27)$$

a value to be compared with the well known estimate from soft pion physics [22]

$$V_{\sigma \pi \pi}(m_\pi^2, m_\pi^2, m_\sigma^2) = m_\sigma \frac{m_\sigma}{2F_\pi} \left( 1 - \frac{m_\pi^2}{m_\sigma^2} \right) = (2.8)m_\sigma . \quad (2.28)$$

Two-pion loops contribute to the  $\pi\pi$  elastic scattering amplitude and appear as corrections to the leading contact term [22,30]. As a further check we may evaluate these corrections in terms of an effective  $\sigma$ -meson exchange in s, u and t-channels. By doing so it is straightforward to show that these corrections are

$$T_{\pi\pi \rightarrow \pi\pi}^{loop} = \frac{2}{m_\sigma^2 - s} V_{\sigma\pi\pi}^2(m_\pi^2, m_\pi^2, s) \delta^{ab} \delta^{cd} + \frac{2}{m_\sigma^2 - u} V_{\sigma\pi\pi}^2(m_\pi^2, m_\pi^2, u) \delta^{ad} \delta^{bc} + \frac{2}{m_\sigma^2 - t} V_{\sigma\pi\pi}^2(m_\pi^2, m_\pi^2, t) \delta^{ac} \delta^{bd}, \quad (2.29)$$

where  $\delta^{i,j}$  is the Kroneker  $\delta$  and with  $i, j$  being pion isospin indices. The factor of 2 is due to the symmetry of the two  $\sigma\pi\pi$  vertices. At threshold this expression amounts to about 20% of the contact term, in good agreement with the value of 25% reported by Gasser [30] from improved low energy theorems.

Now that the  $\sigma\pi\pi$  vertex parameters are well defined we may examine offshellness effects. To this aim we evaluate  $T_{\pi^0 p \rightarrow \pi^0 p}$  at the production threshold of  $pp \rightarrow pp\pi^0$  and compare with its on mass shell value. When both pion legs are on the mass shell the last term in Eqn.2.16 reduces to,

$$T_{\pi^0 p \rightarrow \pi^0 p}^{(t)}(k^2 = m_\pi^2; q^2 = m_\pi^2) = -i \frac{g_{\sigma NN}}{m_\sigma} \left[ g_{0\sigma} + 2 \left( \frac{m_\pi}{m_\sigma} \right)^2 g_{1\sigma} \right] f_\sigma(0) \approx -i 0.11 \frac{g_{\sigma NN}}{m_\sigma}. \quad (2.30)$$

At threshold of the  $pp \rightarrow pp\pi^0$  reaction, the momentum squared of the  $\sigma$  meson is  $(q - k)^2 = (p' - p)^2 \approx -Mm_\pi$  and the off mass shell t pole term in Eqn. 2.16 becomes,

$$T_{\pi^0 p \rightarrow \pi^0 p}^{(t)} = -i g_{\sigma NN} \left( \frac{m_\sigma}{m_\sigma^2 + Mm_\pi} \right) f_\sigma(-Mm_\pi) \left[ g_{0\sigma} + \left( \frac{m_\pi}{m_\sigma} \right)^2 g_{1\sigma} - \frac{Mm_\pi}{m_\sigma^2} (g_{1\sigma} + g_{2\sigma}) \right] \approx -i 1.0 \frac{g_{\sigma NN}}{m_\sigma}, \quad (2.31)$$

which is a factor  $\approx 9$  larger compared to the on mass shell value Eqn. 2.30. In Fig. 5 the amplitude  $T_{\pi^0 p \rightarrow \pi^0 p}$  is drawn as a solid line *vs.*  $q^2$ . The contributions from the s and u channel nucleon pole and  $\sigma$ -meson pole terms are drawn as dashed and dot-dashed curves, respectively. Off mass shell behavior of the s and u terms in Eqn. 2.16 is given by the pion form factor,  $f_\pi(q)$ , the nucleon propagators and the momentum dependence of the  $\pi NN$  vertex. That of the t-channel is affected by the  $\sigma$  meson form factor  $f_\sigma(q - k)$  and the  $V_{\sigma\pi\pi}$  vertex function, Eqn. 2.15. Note that on mass shell both of these contributions are small, opposite in signs and cancel to large extent. As  $q^2$  and  $(q - k)^2$  become more negative both terms become negative and therefore add constructively to the conversion amplitude, giving rise to strongly enhanced off mass shell amplitude. We expect then that  $\pi$  exchange plays an important role in the  $\pi^0$  production process.

### C. The $T_{\eta p \rightarrow \pi^0 p}$ amplitude

The production of a neutral pion can occur also on an internal  $\delta$ -meson line. To evaluate the amplitude for the conversion process  $\eta p \rightarrow \pi^0 p$  we apply a similar procedure as above. The  $\eta$  meson couples to the nucleon isobar  $N^*(1535 \text{ MeV})$  strongly [31] and the main contribution to the conversion amplitude for  $\eta p \rightarrow \pi^0 p$  is due to s and u isobar pole terms (see Fig. 6), hereafter we refer to as the resonance contribution. The other terms (graphs 6c-6e) furnish a background term. A t-channel would involve a vertex with  $\delta$ ,  $\eta$  and  $\pi$  legs defined to be,

$$V_{\delta\eta\pi}(k, q) = m_\delta \left[ g_{0\delta} + \frac{k^2}{m_\delta^2} g_{1\delta} + \frac{q^2}{m_\delta^2} g_{2\delta} + \frac{(k - q)^2}{m_\delta^2} g_{3\delta} \right]. \quad (2.32)$$

Here, to account for the fact that the three legs are different the vertex is described in terms of four constants. Taking the sum of all the graphs in Fig. 6, we may write,

$$\begin{aligned}
T_{\eta p \rightarrow \pi^0 p} = & \\
& -i g_{\eta NN^*} g_{\pi NN^*} f_\eta(q) f_\pi(k) \bar{u}(p') \left[ \frac{1}{M_R - \sqrt{s} + i\Gamma/2} + \frac{1}{M_R - \sqrt{u} + i\Gamma/2} \right] u(p) \\
& i \frac{2M f_{\pi NN}}{m_\pi} \frac{2M f_{\eta NN}}{m_\eta} f_\eta(q) f_\pi(k) \bar{u}(p') \left[ \not{k} \left( \frac{1}{s - M^2} - \frac{1}{u - M^2} \right) - \frac{1}{M} \right] u(p) + \\
& i g_{\delta NN} \frac{f_\delta(q - k)}{(q - k)^2 - m_\delta^2} V_{\delta \eta \pi}(k, q) \bar{u}(p') u(p) . \tag{2.33}
\end{aligned}$$

Here  $M_R = 1535$  MeV and  $\Gamma = 175$  MeV stand for the mass and width of the isobar resonance, and the coupling constants are  $g_{\eta NN^*} = 2.2$ ,  $g_{\pi NN^*} = 0.8$  [31].

We now apply the Adler's consistency conditions to the background term in Eqn. 33 to write the following relations amongst the unknown  $\delta \eta \pi$  vertex constants (see Eqns. 17-19),

$$g_{0\delta} = \frac{m_\delta}{g_{\delta NN} f_\delta(0)} \frac{\sigma_{\pi N \rightarrow \eta N}(0)}{F_\pi F_\eta} , \tag{2.34}$$

$$g_{0\delta} = - \left( \frac{m_\eta}{m_\delta} \right)^2 (g_{2\delta} + g_{3\delta}) , \tag{2.35}$$

$$g_{0\delta} = - \left( \frac{m_\pi}{m_\delta} \right)^2 (g_{1\delta} + g_{3\delta}) , \tag{2.36}$$

where  $F_\eta$  is the  $\eta$  radiative constant; in the limit of an exact SU(3) symmetry  $F_\eta = F_\pi$ . Likewise, the quantity  $\sigma_{\pi N \rightarrow \eta N}$  is a  $\sigma$ -term, a quantity related to matrix elements of various quark mass  $q\bar{q}$  in the nucleon,

$$\sigma_{\pi N \rightarrow \eta N}(t) = \frac{\hat{m}}{2} \langle p' | \bar{u}u + \bar{d}d | p \rangle , \tag{2.37}$$

where  $\hat{m} = (m_u + m_d)/2$  is the average of the u and d quark masses and  $t = (p' - p)^2$  the transferred momentum squared. This term can be deduced from the kaon-nucleon (KN)  $\sigma$ -terms and the strange quark mass through [24]

$$\sigma_{\pi N \rightarrow \eta N}(0) = \left( \frac{\hat{m}}{\hat{m} + m_s} \right) \left[ \sigma_{KN}^{(1)}(0) - \sigma_{KN}^{(2)}(0) \right] . \tag{2.38}$$

Taking the quark masses as  $m_u = (5 \pm 2)$  MeV,  $m_d = (9 \pm 3)$  MeV,  $m_s = (175 \pm 55)$  MeV, and the KN  $\sigma$ -term [24]  $\sigma_{KN}^{(1)}(0) \simeq (200 \pm 50)$  MeV,  $\sigma$ -term  $\sigma_{KN}^{(2)}(0) \simeq (140 \pm 40)$  MeV, one obtains,

$$\sigma_{\pi N \rightarrow \eta N}(0) \simeq (2 \pm 1.3) \text{ MeV} . \tag{2.39}$$

Now the partial decay width of the  $\delta$  meson into a  $\pi\eta$  pair is [29]  $\Gamma_{\pi\eta} = 57 \pm 11$  MeV. By calculating this width using the expression Eqn. 32, one obtains a fourth relation amongst the  $\delta \eta \pi$  vertex constants,

$$g_{0\delta} + \left( \frac{m_\pi^2}{m_\delta^2} \right) g_{1\delta} + \left( \frac{m_\eta^2}{m_\delta^2} \right) g_{2\delta} + g_{3\delta} = \sqrt{\frac{\Gamma 8\pi}{|k|}} \tag{2.40}$$

Resolving this last relation and the Adler's conditions, Eqns. 2.34 - 2.36, gives

$$g_{0\delta} = 0.05 \pm 0.035 , \quad g_{1\delta} = -5.95 \pm 1.5 , \quad g_{2\delta} = -3.46 \pm 0.1 , \quad g_{3\delta} = 3.30 \pm 0.2 . \tag{2.41}$$

With these constants one finds that at threshold for the  $pp \rightarrow pp\pi^0$  reaction, the conversion amplitude is  $T_{\eta p \rightarrow \pi^0 p} \approx 0.98$  fm. The t-pole contribution amounts to only  $T_{\eta p \rightarrow \pi^0 p}^{(t)} \approx 0.04$  fm. We may thus conclude that a  $\delta$  meson pole in a t-channel contributes very little to  $T_{\eta p \rightarrow \pi^0 p}$  and should play a minor role in the  $\pi^0$  production process.

#### D. The $T_{\rho p \rightarrow \pi^0 p}$ amplitude

We follow van Kolck et al. [17] and limit the  $\rho\omega\pi$  vertex to the form,

$$V_{\rho\omega\pi}(k, q) = -\frac{g_{\rho\omega\pi}}{m_\omega} \epsilon_{\mu\nu\lambda\delta} q^\mu k^\delta \rho^\nu \omega^\lambda \pi, \quad (2.42)$$

where the coupling constant  $g_{\rho\omega\pi} \approx -10$ , a value fairly well established. With this expression, the contribution from a  $\rho\omega$  exchange mechanism to the  $T_{\rho N \rightarrow \pi^0 N}$  is

$$T_{\rho N \rightarrow \pi^0 N}^{(t)} = ig_{\rho\omega\pi} g_{\omega NN} f_\omega(q - k) \frac{k^0}{m_\omega} \frac{1}{(q - k)^2 - m_\omega^2} \left[ \frac{1}{E + M} (\mathbf{q} \times \mathbf{p} + i\mathbf{p}\mathbf{q} \cdot \boldsymbol{\sigma} - i\sigma\mathbf{q} \cdot \mathbf{p}) - \frac{1}{E' + M} (\mathbf{q} \times \mathbf{p}' + i\mathbf{p}'\mathbf{q} \cdot \boldsymbol{\sigma} - i\sigma\mathbf{q} \cdot \mathbf{p}') \right]. \quad (2.43)$$

The s and u nucleon pole terms can be calculated from Eqns. 10 and 14.

### III. S-WAVE AMPLITUDES AND CROSS SECTION

In this section we write the primary production amplitude in a form suitable for numerical integration. We call

$$\boldsymbol{\Pi}_j = \frac{\mathbf{p}_j}{E_j + M} \quad (3.1)$$

where  $\mathbf{p}_j$  and  $E_j$  are the three momentum and total energy of the j-th nucleon. For the incoming particles in the center of mass system (CM)  $\boldsymbol{\Pi}_1 = -\boldsymbol{\Pi}_2 = \boldsymbol{\Pi}$ . The energy available in the CM system is

$$Q = \sqrt{s} - 2M - m_\pi, \quad (3.2)$$

where  $s = (p_1 + p_2)^2$  is the total energy squared. We shall calculate the amplitudes and cross sections as functions of the variable  $Q$ . We also define

$$\begin{aligned} q^2 &= -M(m_\pi + Q), \\ a &= 1 + \frac{Q}{m_\pi} \left(1 + \frac{m_\pi}{M}\right), \\ b &= 1 + \frac{Q}{2M} \left(1 + \frac{m_\pi}{2M} + \frac{m_\pi Q}{4M^2}\right), \end{aligned} \quad (3.3)$$

$$R = \frac{1}{M_R - M - m_\pi - Q + i\Gamma/2} + \frac{1}{M_R - M + m_\pi + Q + i\Gamma/2}. \quad (3.4)$$

Following the discussion in the previous section we write the primary amplitude as

$$M^{(in)} = M_\pi + M_\eta + M_\sigma + M_\rho + M_\omega + M_\delta, \quad (3.5)$$

where  $M_B$  stands for a partial production amplitude representing the contribution from the exchange of a boson B. We substitute equivalent two-component free spinor matrix elements (Table A6.1 of Ref. [26]) in Eqns. 2.11-2.13 for s-channels and the appropriate ones for u-channels, to write the scalar and vector meson exchange amplitudes as,

$$M_S = -i \frac{f_{\pi NN}}{m_\pi} \frac{g_{SNN}^2}{m_S^2 - q^2} f_S^2(q) \frac{m_\pi + 2Q}{M + Q} \frac{M}{2M + m_\pi} \boldsymbol{\Pi} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2), \quad (3.6)$$

and

$$M_V = i \frac{f_{\pi NN}}{m_\pi} \frac{g_{VNN}^2}{m_V^2 - q^2} f_V^2(q) [X \boldsymbol{\Pi} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) + iY \boldsymbol{\Pi} \cdot \boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2] \quad (3.7)$$

where we have used the notation

$$\begin{aligned} X &= \left[ 2 \left( 1 + \frac{3m_\pi}{2M} \right) + \kappa_V \left( 1 + \frac{m_\pi + Q}{2M} \right) \left( 1 - \frac{m_\pi + Q}{2(2M + m_\pi)} \right) \right] (1 - \kappa_V \boldsymbol{\Pi} \cdot \boldsymbol{\Pi}) \\ &\quad - \left[ \frac{3m_\pi + Q}{M} + \kappa_V \frac{m_\pi + Q}{2M} \left( 1 - \frac{m_\pi}{2(2M + m_\pi)} \right) \right] (1 + \kappa_V \boldsymbol{\Pi} \cdot \boldsymbol{\Pi}) \end{aligned} \quad (3.8)$$

$$Y = -2 - 2(1 + \kappa_V) \left[ \left( \frac{3m_\pi + Q}{M} \right) + \kappa_V \left( \frac{m_\pi + Q}{2M} \right) \left( 1 - \frac{m_\pi}{2(2M + m_\pi)} \right) \right]. \quad (3.9)$$

The  $\rho\omega$  exchange mechanism contribute to the production amplitude a term,

$$M_\rho^{(t)} = 2 \frac{g_{\rho NN}}{m_\rho^2 - q^2} f_\rho^2(q) 2 \frac{g_{\omega NN}}{m_\omega^2 - q^2} f_\omega^2(q) g_{\rho\omega\pi} \frac{m_\pi}{m_\omega} (\mathbf{p} \cdot \mathbf{\Pi}) \mathbf{\Pi} \cdot \sigma_1 \times \sigma_2. \quad (3.10)$$

To avoid double counting we add this to  $M_\rho$  (but not to  $M_\omega$ ) only. For both of the  $\pi$  and  $\eta$  exchange amplitudes, there is a t-channel pole in addition to s and u-channel nucleon pole terms. The sum of all three gives,

$$M_\pi = -i f_{\pi NN} \left( \frac{2M}{m_\pi} \right) \left( \frac{1}{m_\pi^2 - q^2} \right) f_\pi(q) \\ \left[ f_{\pi NN}^2 \left( \frac{1}{m_\pi} \right) \left( \frac{a}{b} \right) f_\pi(q) + g_{\sigma NN} \left( \frac{1}{m_\sigma^2 - q^2} \right) f_\sigma(q) V_{\sigma\pi\pi}(k^2 = m_\pi^2; q^2) \right], \quad (3.11)$$

$$M_\eta = -i f_{\pi NN} \left( \frac{2M}{m_\eta} \right) \left( \frac{1}{m_\eta^2 - q^2} \right) f_\eta(q) \\ \left[ g_{\eta NN^*} g_{\pi NN^*} R + f_{\pi NN} f_{\eta NN} \left( \frac{f_\eta(q)}{m_\eta} \right) \left( \frac{a}{b} \right) + g_{\delta NN} \left( \frac{f_\delta(q)}{m_\delta^2 - q^2} \right) V_{\delta\eta\pi}(k^2 = m_\pi^2; q^2) \right]. \quad (3.12)$$

Finally, the total cross section is calculated from the expression,

$$\sigma = \frac{M^4}{16(2\pi)^5 \sqrt{(s)} \mathbf{p}_1} \int \frac{d^3 \mathbf{p}_3}{E_3} \frac{d^3 \mathbf{p}_4}{E_4} \frac{d^3 \mathbf{p}_\eta}{E_\eta} |Z M^{(in)}|^2 \delta^4(p_i - p_f), \quad (3.13)$$

where  $Z$  is the three-body FSI correction factor of Refs. [20] to be specified below.

#### IV. PREDICTIONS AND COMPARISON WITH DATA

We apply now the model presented in the previous sections to calculate the total cross section for the  $pp \rightarrow pp\pi^0$  reaction at energies close to threshold. We first consider the relative importance of the various exchange contributions. To this aim we draw in Fig. 7 the primary production amplitude  $M^{(in)}$  along with the partial exchange amplitudes of Eqns. 46-51, vs. the energy available in the center of mass (CM) system. The main contribution is due to pion exchange with ratios  $M_\pi : M_\rho : M_\omega : M_\sigma : M_\eta : M_\delta \approx 138 : 42 : 8 : 6 : 5 : 0.6$ . Next important to the pion is contribution from the  $\rho$  meson ( $M_\rho^{(s+u)} \approx 30\%$  and  $M_\rho^{(t)} \approx 4.5\%$  of  $M^{(in)}$ ). In comparison with  $M_\pi$  other contributions are significantly smaller. Nonetheless, they influence the cross section strongly through interference. They all have a common relative phase and add constructively. The  $\rho$ , having an opposite sign counteracts to balance their effects.

The ratios quoted above differ considerably from those predicted by Horowitz et al. [8]. Particularly, the s and u-channel nucleon pole terms for scalar meson exchanges have opposite signs, thus suppressing the  $\sigma$  exchange contribution and practically eliminating that from the  $\delta$  meson. We note however, that as in Ref. [8] the nucleon pole term contributions from  $\pi$  and  $\sigma$  exchanges add constructively.

We want to emphasize at this stage that, our transition operator  $M^{(in)}$  for the production process, involves contributions from connected diagrams (Fig. 3) only, and accounts for all relativistic and crossing symmetry effects. Taking u-channel contributions only, one obtains far more important effects than with both of the s and u-channels. The lesson to be learned here is that many small terms added up coherently can explain a seemingly large discrepancy, at a time when other small terms which counteract to balance their effects are disregarded. Under these circumstances it seems essential to preserve crossing symmetry at all stages of the calculations and treat all contributions on an equal basis.

It is rather surprising that the  $\sigma$  contribution is not as important as predicted in Ref. [8]. Yet, it is to be indicated that in Ref. [8], only contributions from negative-energy intermediate states are included explicitly to the pion production operator via a Z-graph, while contributions from positive-energy nucleon intermediate state are presumably contained in the distorted waves describing the initial and final two-nucleon states. Thus, different approximations are used in the calculations of the direct and Z-graph from either s or u-channels and it is not clear how a delicate balance between these contributions is maintained throughout the calculations.

In Fig. 8 we draw predictions for the total  $pp \rightarrow pp\pi^0$  cross section as obtained for three different value of the  $\pi N$   $\sigma$ -term. The small dashed curve shows results obtained without the  $\sigma$ -meson pole term included. Clearly, the overall

contribution from nucleon pole terms only does not provide the enhancement required to resolve discrepancy between the calculated cross section and data. Albeit, the contribution from an isoscalar-scalar  $\sigma$ -meson pole term dominates the production cross section and as shall demonstrate below, with a  $\sigma_{\pi N}(0) = 35 \text{ MeV}$ , the  $\sigma$  pole term provides the enhancement required to explain cross section data near threshold.

The strong dependence of our predictions on the  $\pi N$   $\sigma$ -term deserve a comment. Though still not very well known, it seems more likely that the value of this quantity falls in the range  $\sigma_{\pi N}(0) = 35 - 45 \text{ MeV}$  [27]. In the calculations presented in the present work we have not included contributions from  $\Delta(1232 \text{ MeV})$  isobar excitations. It remains still to be verified whether the effects of such term would bring the calculated cross section to agree with data even with a larger value of  $\sigma_{\pi N}(0)$ .

## V. FSI AND COMPARISON WITH DATA

In comparison with data, the predicted cross section in Fig. 8 varies very fast with energy due to phase space factor. The primary production amplitude (see Fig. 7) is practically constant near threshold and therefore contributes very little to the energy dependence. To cure this deficiency of our predictions and allow for comparison with data to be made we must account for ISI and FSI effects. In what follows we treat ISI and FSI using two different approximations, which as we shall demonstrate below both yield similar cross sections.

### Approximation I

In this approximation the transition operator  $M^{(in)}$  is treated as an effective operator acting on nucleon wave functions. These are calculated using a phenomenological NN potential. In this approximation we neglect interactions of the  $\pi^0$  produced with the outgoing nucleons. This is a standard and usual procedure in the literature [3,13], reducing a three-body problem into effectively a two-body process. To be consistent with the OBE picture applied in the present work, we use the OBE NN potential of Machleidt [19]; potential parameter set C of his table A.2. The initial and final two-nucleon radial wave functions are calculated in momentum space from half off shell  $R$  matrix, following exactly the procedure of Ref. [19]. The evaluation of the transition amplitude and cross section is performed as in Ref. [13]. We have verified that the calculated wave functions reproduce well the known experimental phase shifts [33].

### Approximation II

Here the S-wave production amplitude is assumed to factorize into a primary production amplitude  $M^{(in)}$ , a P-wave ISI factor and an S-wave FSI factor [20],

$$T_{2 \rightarrow 3} = \langle \Psi_{el,f}^{(+)(3)} | M_{2 \rightarrow 3}^{(in)} | \Psi_{el,i}^{(-)(2)} \rangle \approx Z_{33} M_{2 \rightarrow 3}^{(in)} X_{22} , \quad (5.1)$$

where  $\Psi_{el,i}^{(-)(2)}$  and  $\Psi_{el,f}^{(+)(3)}$  represent the initial two-body and final three-body wave functions. The ISI correction factor is taken to be  $X_{22} \approx |1 + \sin(\delta_{3P_0}) \exp i(\delta_{3P_0})|$ . Here  $\delta_{3P_0}$  is the pp P-wave phase shift. At threshold of the  $pp \rightarrow ppp\pi^0$  reaction,  $\delta_{3P_0} \approx -10^\circ$  so the ISI corrections to the cross section are bound to  $\approx \pm 30\%$ . For a three body process as in our case, the FSI correction factor is identified [20] with the elastic scattering amplitude (on mass shell) for the process  $\pi NN \rightarrow \pi NN$  (three particles in to three particles out) and has the structure of the Faddeev decomposition of the t-matrix for  $3 \rightarrow 3$  transition. An important property of this approximation is that the different two body interactions among the out going particles contribute coherently. Although the meson-nucleon interactions are weak with respect to the NN interaction, they can still be influential through interference. This approximation was discussed in length elsewhere [20] and we skip further details here. In the analysis presented below the FSI factor is estimated from  $\pi N$  and NN elastic S-wave scattering phase shifts [34,35]. The S-wave NN phase shift is obtained from the effective range expansion which includes Coulomb interaction between the two protons. We have used the scattering length  $a_{pp} = -7.82 \text{ fm}$  and an effective range  $r_{pp} = 2.7 \text{ fm}$  of Ref. [34]. The S11 and S13  $\pi N$  scattering lengths are taken to be  $a_1 = 0.173 \text{ m}_\pi^{-1}$  and  $a_3 = -0.101 \text{ m}_\pi^{-1}$ , respectively [35].

To compare the results from the two approximations we draw in Fig. 9 the partial cross sections corresponding to production via a  $\sigma$  meson t pole mechanism. For neutral pion production, the cross sections calculated using the two approximations are practically identical. We note though that Approximation I does not account for final meson-nucleon interactions. To see how influential the  $\pi N$  FSI can be, we draw in Fig. 10 the cross section corrected for FSI interactions assuming pure  $I=\frac{1}{2}$  and  $I=\frac{3}{2}$  interactions for the outgoing  $\pi^0 N$  pairs. The energy dependence of the cross section differ significantly for the two channels. But, when the  $\pi N$  interactions are taken in the proper isospin combination, their overall contribution to the FSI factor almost cancels out, leading to an energy dependence practically identical with the one obtained with charged pp FSI (small dash curve) only. We want to stress here

that this is an accidental consequence of the fact that the  $\pi^0 p$  interaction over the allowed  $I=1/2$  and  $I=3/2$  isospin channels averages to zero, making contributions from diagrams with  $\pi^0 p$  interactions ineffective. This may not be the case for charged pions and  $\eta$  meson production [20].

## VI. SUMMARY AND CONCLUSIONS

We have calculated S-wave pion production in  $pp \rightarrow pp\pi^0$  using a covariant OBE model, where a boson B created on one of the incoming protons is converted into a neutral pion on the second. The amplitude for the conversion process  $BN \rightarrow \pi^0 N$  are taken to be the sum of s and u pole terms and when allowed a meson pole term in a t-channel. To be consistent with the OBE picture of the NN interaction we have considered contributions from all of the  $\pi$ ,  $\eta$ ,  $\sigma$ ,  $\rho$ ,  $\omega$  and  $\delta$  mesons. Both the scale and energy dependence of the cross section are reproduced rather well.

Based on a covariant quantum field theory, the transition operator must be fully covariant and as such it includes contributions from irreducible connected diagrams only, each involving large momentum transfer. The operator  $M^{(in)}$ , Eqn. 48, and likewise the calculated amplitude preserve crossing symmetry and relativistic covariance throughout the calculations. Furthermore, terms which involve three meson vertices are written in a general form. As an example, the Lagrangian corresponding to a  $\sigma\pi\pi$  vertex, in addition to non derivative terms, includes derivative  $\sigma(\partial\pi\partial\pi)$  and  $(\partial\sigma\partial\pi)\pi$  interaction terms.

The off-shell properties of the various partial production amplitudes are taken into account explicitly. For vector and scalar mesons, offshellness is contained in the nucleon form factor and has a marginal influence. A pseudovector coupling for the pseudoscalar particles, as assumed in this work, introduces an extra factor of  $q$  to the amplitude. Consequently for the  $\pi$  and  $\eta$ , each of the s and u nucleon pole terms at  $q^2 = -3.3 fm^{-2}$  becomes a factor of  $\approx 7$  higher in comparison with its value on mass shell. To account for the off mass shell behavior of the  $\sigma$ -meson pole term, we have used Adler's consistency conditions and the isospin even  $\pi^0 p$  scattering length to write the  $\sigma\pi\pi$  vertex in a rather general form. Taking all of the s, u and t-channel contributions into account, we have found that the  $\pi^0 p \rightarrow \pi^0 p$  conversion amplitude off the mass shell is more than a factor  $\approx 15$  higher in comparison with its on mass shell value. Consequently, the  $\pi$  exchange contribution to the amplitude for the  $pp \rightarrow pp\pi^0$  reaction dominates the production process and brings the calculated cross section to agree with data.

We have found that the contribution from a  $\sigma$ -meson pole in a t-channel dominates pion production at threshold. A term as such accounts effectively for two-pion exchange contributions. The  $\sigma\pi\pi$  vertex used to calculate this contribution depends rather strongly on the  $\pi N$   $\sigma$ -term. However, even if we use a value,  $\sigma_{\pi N}(0) = 25 MeV$ , which is unrealistically too low, a t-channel  $\sigma$  pole term amounts to more than a factor of two higher a contribution than that of the s and u-channel nucleon poles. It seems quite impossible to disregard such an important contribution to the production process. Neither the overall contribution from all meson exchanges, nor any of the various meson exchange contributions, from s and u nucleon pole terms only, can reproduce data for this process. In fact, various contributions of a similar size are found to interfere destructively and including all of these in a consistent manner, yields a production amplitude far below what would be required to explain data. This conclusions seems unavoidable and independent on the approximation used to account for ISI and FSI corrections.

A meson production in NN collisions necessarily involves large momentum transfer. Thus, two-pion exchanges which describe short range interactions are expected to play an important role. In a traditional covariant OBE model such contributions are represented by an effective scalar  $\sigma$ -meson pole term. In  $\chi$ PT calculations these should result from expanding the existing calculations to include one loop two-pion diagrams. Recently, we have extended the existing  $\chi$ PT calculations [11–13] taking into account all tree and loop diagrams up to chiral order  $D=2$ . With these one loop contributions added, it is found that in analogy with a dominant  $\sigma$  meson pole term, there are substantial contributions from isoscalar-scalar two-pion t-channel exchanges [36].

In summary, two-pion exchanges play an important role in the production of  $\pi^0$  in  $pp \rightarrow \pi^0 pp$ . By taking these into account, we have demonstrated that a covariant OBE model can explain the existing near threshold neutral pion production data .

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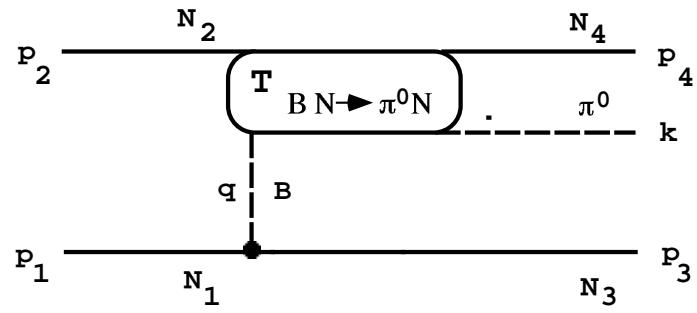


FIG. 1. The primary production mechanism for the  $NN \rightarrow NN\pi^0$  reaction. In this figure and following figures, solid lines represent nucleons and broken lines mesons.

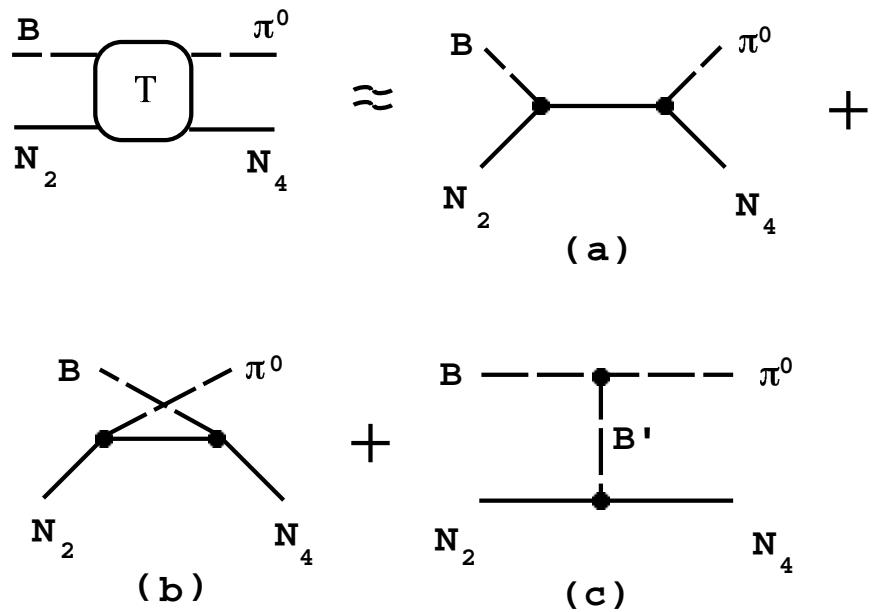


FIG. 2. Graphs contributing to the amplitude of the conversion process  $BN \rightarrow \pi^0 N$ . Graphs a and b stand for nucleon pole in s and u-channel; the graph c represents a  $B'$  meson pole in t-channel.

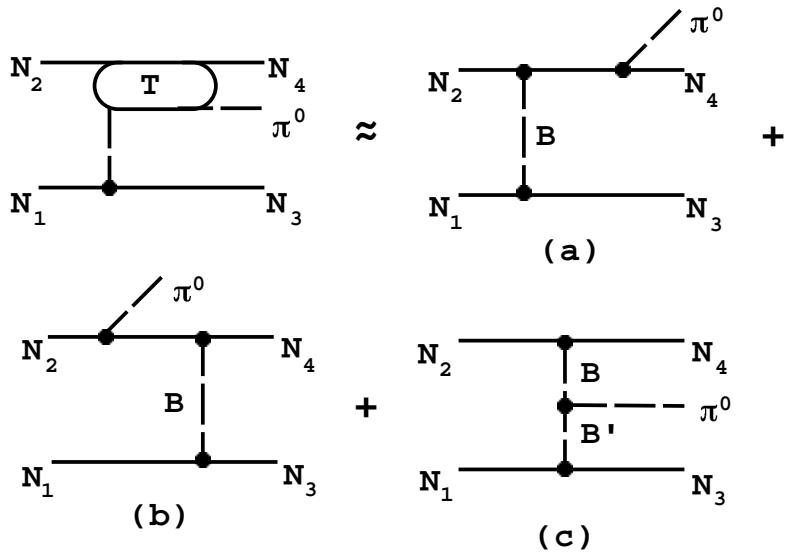


FIG. 3. The production amplitude. The mechanisms of graphs a and b represent pion production on external nucleon lines. Graph c represents production on an internal meson line. From parity and isospin conservations the latter mechanism is limited to production from  $\sigma$  and  $\delta$  meson lines only.

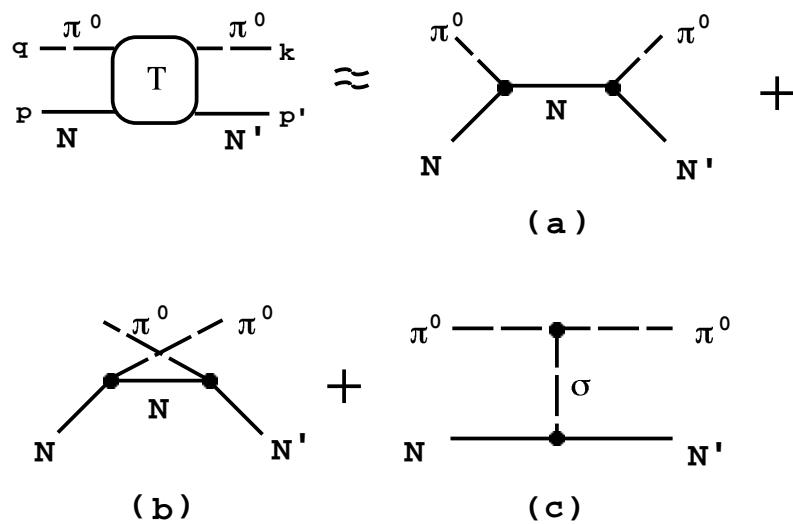


FIG. 4. Terms contributing to the  $\pi^0 p \rightarrow \pi^0 p'$  conversion amplitude. Graph c describes an effective isoscalar  $\sigma$ -meson pole in a t-channel.

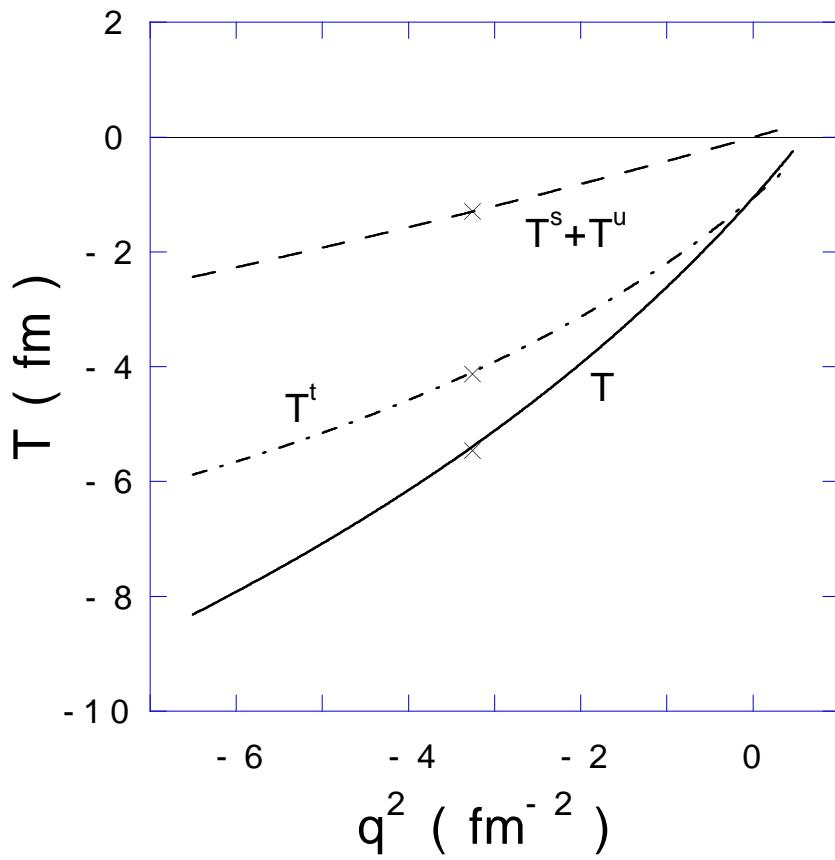


FIG. 5. The off mass shell behavior of the  $T_{\pi^0 p \rightarrow \pi^0 p}$  amplitude. The contribution from nucleon s and u-channel pole terms and that from a  $\sigma$ -meson pole are drawn as dashed and dot-dashed curves, respectively. The crosses point threshold of the  $pp \rightarrow pp\pi^0$

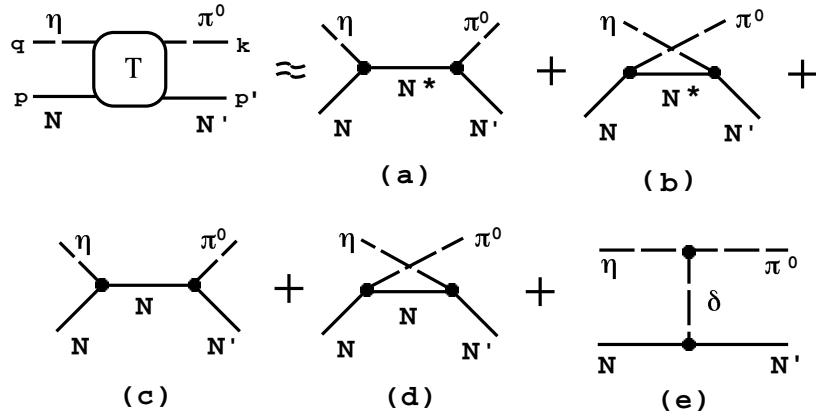


FIG. 6. Terms contributing to the conversion amplitude for the  $\eta p \rightarrow \pi^0 p$ . Because of the strong coupling between the  $\eta$ -meson and the  $N^*$  (1535 MeV) nucleon isobar one should allow for contributions from nucleon and nucleon isobar intermediate states. Graph e describes  $\delta$ -meson pole in a t-channel.

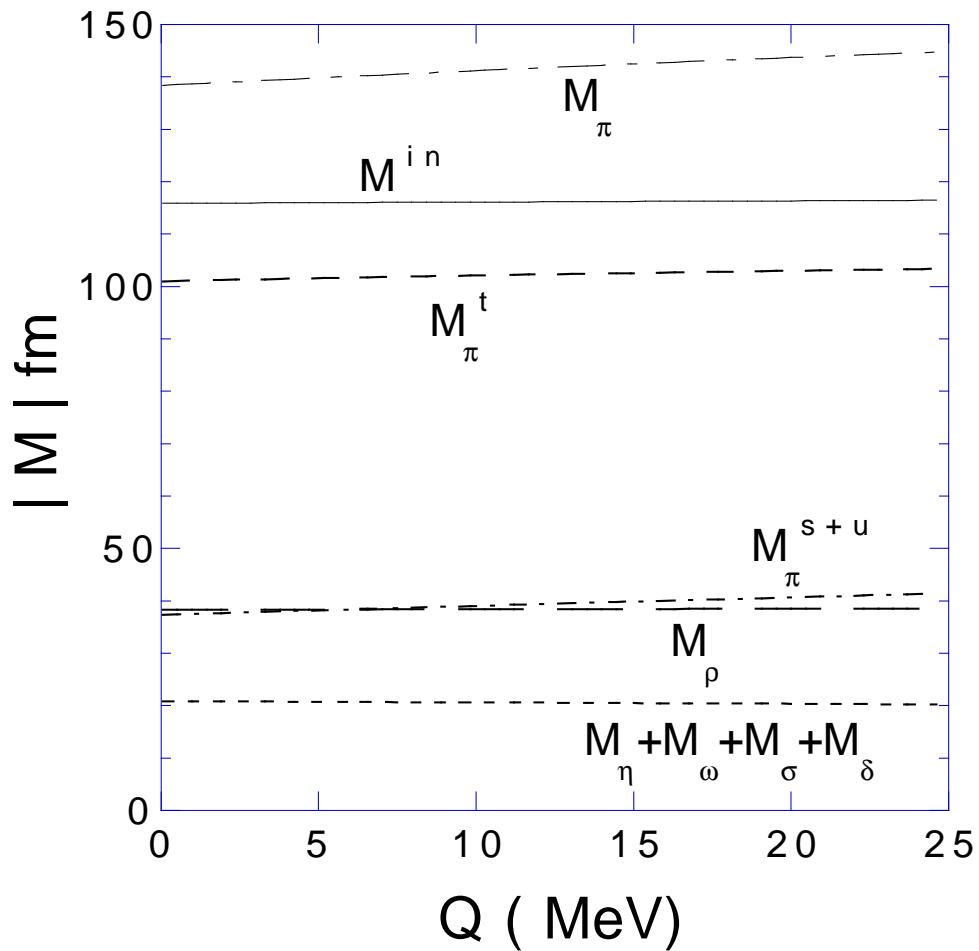


FIG. 7. The primary production amplitude for the  $pp \rightarrow pp\pi^0$  reaction vs.  $Q$ , the energy available in the overall center of mass system. Different exchange contributions to  $M^{(in)}$  are drawn separately.

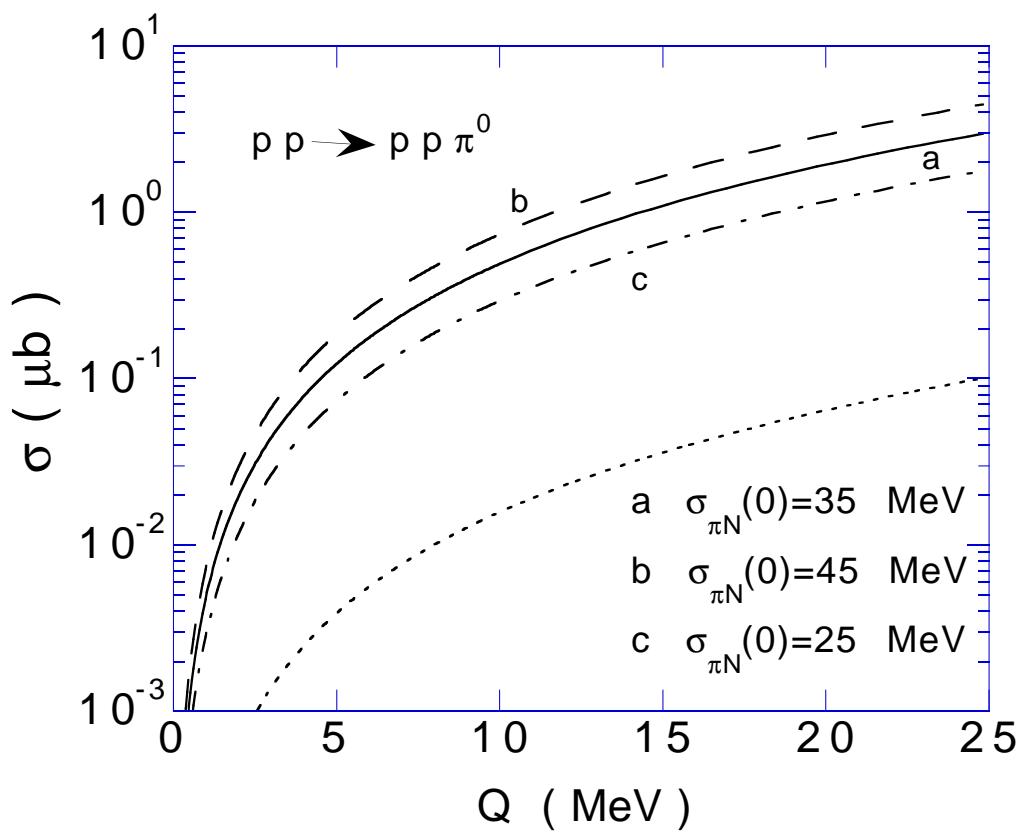


FIG. 8. Predictions for the total cross section of the  $pp \rightarrow pp\pi^0$  reaction for different values of  $\sigma_{\pi N}(0)$ . Predictions with s and u nucleon pole terms only are shown by the dotted curve. All curves are not corrected for FSI.

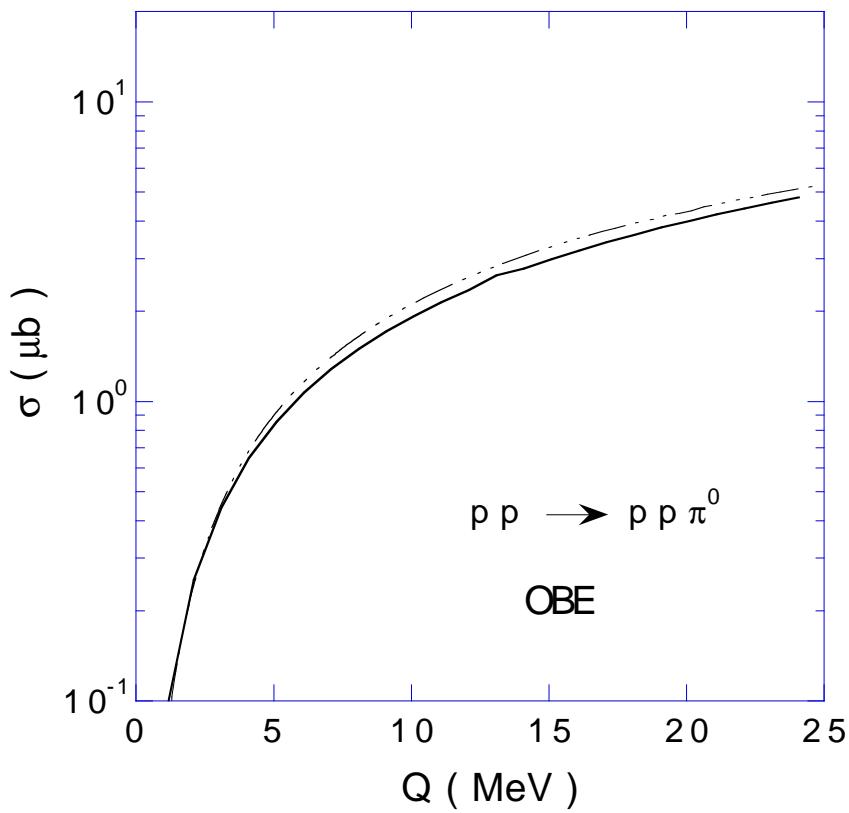


FIG. 9. ISI and FSI corrections. Energy integrated cross sections assuming production via a  $\sigma$  meson t pole mechanism only with Approximation I (solid line) and Approximation II (dot-dashed line)

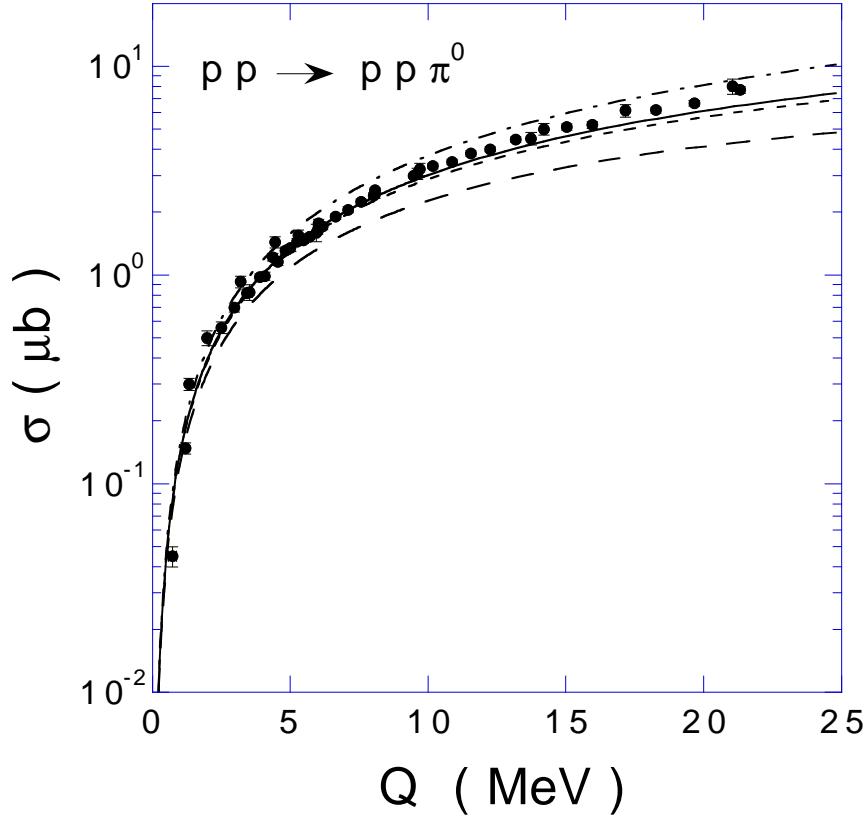


FIG. 10. FSI corrections. Integrated energy cross sections calculated with the assumption that the interacting  $\pi N$  pair is scattered in isospin  $I=\frac{1}{2}$  (large dashed curve) and  $I=\frac{3}{2}$  (dot-dashed curve). The solid line is that obtained with the  $\pi N$  interactions taken in the appropriate isospin combinations. Predictions which account for the pp FSI only (small dashed curve) are nearly identical with the solid curve. All predictions are calculated with  $\sigma_{\pi N}(0) = 35$  MeV. The data points are taken from Refs. [1,2]